Discussion

It was shown earlier[4] that ignition of a composite ammonium perchlorate propellent can occur rapidly at the decomposition temperature of ammonium perchlorate and that it is not necessary for the perchloric acid-fuel mixture to become further heated by diffusing from the propellent surface into the hotter igniter gases. However, it was not possible to deduce whether the important exothermic reactions, which lead to ignition, occur between perchloric acid vapor and the condensed fuel or vaporized fuel.

In the present work two approaches to the solution of this question have been made. First, ignition studies have been carried out with less volatile fuels (carbon, nylon, and terylene), and these have been found to ignite readily. Second, a study has been made of the ignition characteristics of the gaseous species expected to be formed during the ignition phase of an ammonium perchlorate–polyisobutene propellent (i.e., isobutene, methane, ethylene, and ammonia) and of the effect of surfaces and catalysts upon these ignition characteristics.

It has been shown that the ignition of gaseous fuels with perchloric acid vapor in the absence of a surface is considerably slower than that of the condensed fuel[4] which would form these gaseous fuels by pyrolysis. Consequently, this suggests that heterogeneous reactions are the important ones leading to ignition in a propellent containing ammonium perchlorate as an oxidizer.

The ignition of a composite ammonium perchlorate propellent can now be considered in more detail. As the temperature rises, the ammonium perchlorate will vaporize to ammonia and perchloric acid, and the fuel will pyrolyze to give a mixture of simple hydrocarbons and fuel monomer. There are several possible reactions by which such a system can proceed to ignition and these are outlined: 1) ignition by gaseous reactions, e.g., perchloric acid–ammonia (or monomer, or simple hydrocarbon); 2) ignition by gaseous reactions in the presence of a catalytic surface, e.g., perchloric acid–ammonia (or monomer)–catalyst; 3) ignition by heterogeneous reactions; e.g., perchloric acid–solid fuel (or ammonium perchlorate). A fourth possible route to ignition is by exothermic reactions occurring in the solid.[6] This route is not considered further.

It is to be emphasized that groups 2 and 3 include the heterogeneous decomposition of perchloric acid vapor to chlorine oxides, which may subsequently react either homogeneously or heterogeneously with fuel. It has been shown that ignition by reactions in group 1 is much slower at 200°–300°C than in groups 2 and 3. In group 2 it has been shown that the perchloric acid–ammonia–catalyst system is much faster to ignite than the perchloric acid–gaseous hydrocarbon–catalyst system. In group 3 the reaction perchloric acid–solid fuel is much faster under the experimental conditions used than is perchloric acid–ammonium perchlorate.

It was found experimentally that perchloric acid–ammonia–catalyst mixtures ignited faster than perchloric acid–solid fuel mixtures. Since in the practical case of an ammonium perchlorate propellent the ammonia and perchloric acid are vaporized together, it seems probable that, with the catalyzed propellent, ignition occurs as a result of heterogeneous reactions between the ammonia and perchloric acid vapor on the surface of the catalyst. The alternative route would involve diffusion of the perchloric acid vapor to the fuel surface before heterogeneous reaction could occur, and this would appear less probable. In an uncatalyzed propellent it is likely that both perchloric acid–ammonia on the propellent surface and perchloric acid–solid fuel could be the heterogeneous reactions leading to ignition.

The catalysis of the ammonia–perchloric acid ignition by cupric chromate and ferric oxide is in accord with the observations that these substances are effective catalysts in the ignition of propellents.[7,8] The inert nature of titanium dioxide, silica, and alumina is also in agreement with previous work on the ignition of ammonium perchlorate.[9]

The present findings give strong support to the conclusions made by Wise et al.[7] as a result of experiments on the ignition of ammonium perchlorate–copper chromite mixtures in an arc image furnace. They concluded that when ignition occurred by reactions involving gaseous species, the critical species was perchloric acid, and that the copper chromite promoted ignition by catalyzing the decomposition of perchloric acid, which could then react with gaseous fuel.

Wise has also shown[7] that at high heat fluxes, similar to those occurring in practice, higher surface temperatures are involved. It is assumed that the present findings can be extrapolated to this higher temperature range.

In conclusion, the present work suggests that the heterogeneous theory of Anderson et al.[2] is more probable for the ignition of ammonium perchlorate composite propellent in a rocket motor, although the homogeneous gas phase theory of Summerfield et al.[1] is probably valid for ignitions with oxygen gas in a shock tube.

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Tow Cable Loading Functions

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Introduction

THE design of cables to tow or tether underwater bodies has received a considerable amount of attention over a number of years.[1–5] As the desired towing speed and depth of submergence have increased, the drag of the cable has be-

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Received June 8, 1966; revision received July 29, 1966. This work was sponsored under Contract N140(122)-78185B under the technical direction of the U. S. Naval Underwater Weapons Research and Engineering Station, Newport, R. I.

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come most important. Low drag profiles have been employed to reduce the cable tension and also to obtain greater towing depths. The loading functions to be used to describe the fluid dynamic forces on the cables have been questioned continually. It is the purpose of this note to present a set of loading functions that should adequately define the fluid-dynamic forces acting on a low-drag profile cable operating at a Reynolds number that is high enough to insure that the pressure drag is a negligible portion of the total drag.

1. Theory

Figure 1 shows the cable configuration and the forces acting on a cable element for the operating conditions just assumed. Resolving the forces in the directions normal and tangential to the cable element, the expressions are obtained:

$$\frac{dT}{ds} = R\cos\theta + w\sin\theta \qquad \frac{d\theta}{ds} = \frac{w\cos\theta - R\sin\theta}{T}$$

which may be combined to yield

$$\frac{dT}{T} = \frac{R\cos\theta + w\sin\theta}{w\cos\theta - R\sin\theta}\,d\theta \tag{1}$$

where R is the resultant fluid-dynamic force on the cable, which may be a function of θ.

The "loading functions" are usually defined as

$$g(\theta) = R_t/R^* \qquad f(\theta) = R_n/R^* \tag{2}$$

where R_t and R_n are the tangential and normal components of the fluid-dynamic force acting on the cable element, respectively, and R^* is the fluid-dynamic force acting on the cable element when θ equals $\pi/2$.

Substituting Eq. (2) into Eq. (1) yields

$$\frac{dT}{T} = \frac{g(\theta) + (w/R^*)\sin\theta}{(w/R^*)\cos\theta - f(\theta)}\,d\theta$$

Evaluation of this equation depends on the relationship assumed for R as a function of θ.

Eames[4] assumes that R is a constant and equal to R^*, thus, the loading functions are

$$f(\theta) = \sin\theta \qquad g(\theta) = \cos\theta$$

For the low-drag profile at supercritical Reynolds number, the Eames loading functions should be adequate if θ is near $\pi/2$. For high towing velocities and deep submergence of the towed body, however, θ may become quite small at the surface. Also, for tethering cables (Fig. 2) with which extreme submergence depths of the body are considered, θ may be quite small at the surface and may be much larger than $\pi/2$ at the submerged body.

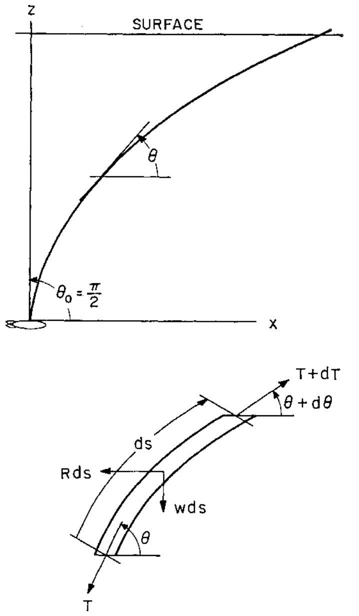

Fig. 1 Towing cable.

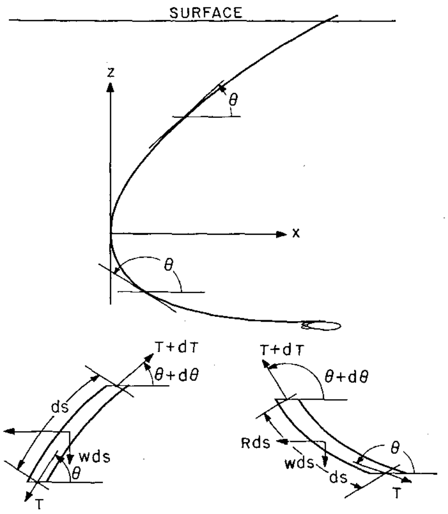

Fig. 2 Tethering cable.

As θ decreases (or increases toward π), the profile apparent thickness distribution changes, and the profile apparent Reynolds number changes. It is not expected that the apparent change in thickness distribution will have a large effect for the assumed conditions; however, the apparent change of Reynolds number should be considered.

If it is assumed that the effect of Reynolds number for the profile is similar to the effect on a flat plate, then

$$C_D/C_D^* = (R_e^*/R_e)^{1/5}$$

where R_e^* is the profile Reynolds number for $\theta = \pi/2$ and R_e is the profile apparent Reynolds number for $\theta \neq \pi/2$. Since $R_e = R_e^*/\sin\theta$ then

$$f(\theta) = \frac{R\sin\theta}{R^*} = \frac{C_D\rho/2U_0^2cds\sin\theta}{C_D^*\rho/2U_0^2cds}$$

or

$$f(\theta) = \sin^{6/5}\theta$$

and, similarly,

$$g(\theta) = \cos\theta\sin^{1/5}\theta$$

2. Comparison with Eames' Loading Functions

If the preceding loading functions are considered as corrections to the Eames loading functions, the correction is 10% when θ has a value of about 36°.

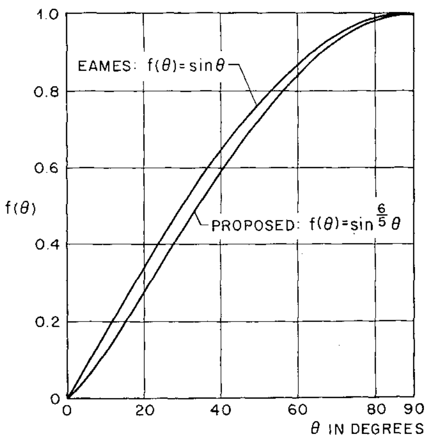

Fig. 3 Comparison of normal loading functions.

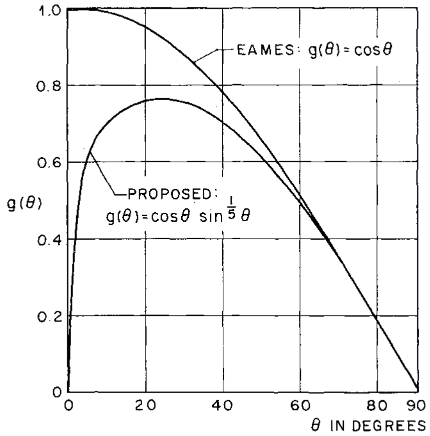

Fig. 4 Comparison of tangential loading functions.

The correction becomes larger as θ decreases. The function $g(\theta)$ equals zero when θ equals zero and has its maximum value at $\theta \doteq 24°$.

The proposed loading functions and the loading functions calculated by Clark,[5] who used experimental wing data, have similar characteristics. Comparisons of the loading functions proposed herein with Eames' loading functions are shown in Figs. 3 and 4.

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A Turbulent-Wake Model with Axial, Laminar Diffusion

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The fluctuations of electron density in turbulent wakes are controlled by several physical mechanisms. Consider a slender body and its turbulent wake preceded by a laminar run. For such bodies, the peak electron concentrations are generated in the boundary layer and are swept into the laminar wake as a rather narrow pulse centered on the wake axis.[1] During transition, the pulse breaks up into blobs of ionized fluid. Then, random convection transports the blobs across the mean enthalpy profile of the turbulent wake. This process creates large, local gradients that are degraded by laminar dissipation. Finally, rate chemistry dictates the response of the ionized blob to its varying environment.

This difficult problem has motivated several approaches[2] that range from mixing-length theory to marble-cake models. More recently, a Monte Carlo method has been developed[3] and applied to simple chemistries.[3–5] The principle is to trace the motion of ionized blobs through a random velocity field that represents the turbulent wake. Velocity samples are produced on the computer. The data then are averaged to yield mean and rms electron densities and correlation function. Laminar diffusion is neglected in the results of Refs. 3–5, although viscous dissipation is accounted for in the specification of the turbulent velocities. Correspondingly, the electron density fluctuations predicted in Ref. 3 become fairly large downstream of transition; depending on flight conditions, the variance of the electron density can exceed the mean electron density. The purpose of this note is to extend the formulation of Ref. 3 by including a dominant mode of laminar diffusion. An analytic solution[6] is derived for a linear electron chemistry.

Before describing the solution, we review briefly the convection model that is applied in Refs. 3–6. It is clear that a detailed specification of the turbulent velocity field is impractical; it exceeds both computer capability and current experimental information. This major difficulty can be reduced in the case of slender bodies. As noted previously, the electron pulse is narrow in the laminar wake. Downstream, we focus our attention on small blobs that wander within the turbulent core.[7] These play a dominant role since both mean and rms electron density are expected to peak in the wake core (by analogy with gas density, velocity, and temperature[7,8]). Hence, the convection model primarily should describe turbulent dispersion near the wake axis. Across this narrow region, the velocity fluctuations remain strongly correlated. Correlation loss occurs mainly along the path of the electron pulse. Since the axial velocity is much larger than the transverse components,[7] such paths are roughly parallel to the wake axis, say OX; thus, the transverse velocity fluctuations can be represented by random functions of X only. The mean flow is described by a series expansion valid near the axis. The mathematical details are discussed in Ref. 3. The basic consistency of the model also is examined; it is shown that the mean electron density profile spreads by turbulent diffusion and attains asymptotically the width of the velocity wake.

For this convection model, laminar diffusion would have two distinct effects. Diffusion transverse to the axis increases the pulse width and depresses the peak density in the expected manner; estimates show that this effect is rather slow below 120 kft alt. Diffusion in the axial direction couples strongly with the velocity fluctuations and may be enhanced considerably. Thus, it is useful to obtain a solution that includes axial diffusion. A closed-form solution is required because a purely numerical treatment raises difficult questions of boundary-conditions and asymptotic behavior.

Let α_e be the electron mass fraction; we consider the basic case of a frozen chemistry. The results can be extended readily to a variable diffusion coefficient $D(X)$. The species equation is

$$\bar{U}_c(X)\,\frac{\partial \alpha_e}{\partial X} + V'(X)\,\frac{\partial \alpha_e}{\partial Y} + W'(X)\,\frac{\partial \alpha_e}{\partial Z} = D\,\frac{\partial^2 \alpha_e}{\partial X^2} \qquad (1)$$

For simplicity, the transverse mean velocity has been neglected. Its effect is shown to be small in Ref. 3. Thus, V' and W' are velocity fluctuations. The fluctuation of axial velocity U' is much smaller than the mean component $\bar{U}$ and need not be included. Because of the narrow pulse assumption, $\bar{U}$ is replaced by its value on the axis, $\bar{U}_c(X)$, and V' and W' are random functions of X only. Finally, the diffusion coefficient D is assumed constant to simplify the algebra. The extension to $D(X)$ is easy. We note that the coordinates (X, Y, Z) include a simplified Howarth-Dorodnitsyn transformation[3] which accounts for the main effect of compressibility.

Received June 9, 1966; revision received June 10, 1966. This work was supported under the ABRES program, AF Contract AF04(694)-890.

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